A way to calculate the probabilities using 10-again rule

Let's throw $n > 0$ dice. Without 10-again rule the chance of getting exactly $0 \leq p \leq n$ successes is clearly

$$T(n,p) = \binom{n}{p} 0,3^p \cdot 0,7^{n-p} \tag{1}$$

and $T(0,0) = 1$.

We want to calculate how to get additional $i \geq 0$ successes from p successes with 10-again rule. Let's define function

$$P(p,i) = \binom{p}{i} (\frac{1}{3})^i \cdot (\frac{2}{3})^{p-i}. \tag{2}$$

It shows what are the chances that there were $0 \leq i \leq p$ 10's in those $p \geq 1$ successes. Now we have all the basic functions we need.

Now the fun part begins. Let's define that j's mean the number of successes and k's mean the number of 10's rolled in those successes. Let assume that

$$a = ((j_1,k_1),(j_2,k_2),\ldots,(j_t,k_t)) \tag{3}$$

is finite sequence of pairs that fulfills the following three conditions:

$$j_1 + j_2 + \ldots + j_t = m, \tag{4}$$

$$n \geq j_1 \geq k_1 \geq j_2 \geq k_2 \geq \ldots \geq j_t \geq k_t \geq 0, \tag{5}$$

and

$$j_t > 0. \tag{6}$$

The condition 4 means that in the end we have to have exactly m successes. The condition 5 limits the number of 10's and followings successes for those combinations that can actually happen. The condition 6 guarantees that the sequences will end if $m \in \mathbb{Z}_+$.

Let A be the set of those sequences a that satisfy three conditions above. Now the probability of getting exactly m successes with n dice using 10-again rule is

$$f(n,m) = \sum_{a \in A} \left(T(n,j_1)P(j_1,k_1)T(k_1,j_2)P(j_2,k_2)\cdots T(k_{t-1},j_t)P(j_t,k_t)T(k_t,0) \right). \tag{7}$$

Now it is "easy" to calculate the probabilities. We just generate the set A and do the sum. For example, if we throw 2 dice and we want to get 2 successes then

$$A = \{\ ((2,2))\ ,\ ((2,1))\ ,\ ((2,0))\ ,\ ((1,1),(1,1))\ ,\ ((1,1),(1,0))\ \}. \tag{8}$$

Thus

$$\begin{aligned} f(2,2) &= T(2,2)P(2,2)T(2,0) + T(2,2)P(2,1)T(1,0) + T(2,2)P(2,0)T(0,0) \\ &\quad + T(2,1)P(1,1)T(1,1)P(1,1)T(1,0) + T(2,1)P(1,1)T(1,1)P(1,0)T(0,0) \\ &= T(2,2)(P(2,2)T(2,0) + P(2,1)T(1,0) + P(2,0)) \\ &\quad + T(2,1)P(1,1)T(1,1)(P(1,1)T(1,0) + P(1,0)) \\ &= 0.1107. \end{aligned}$$

By the way, this isn't supposed to be the most efficient way to do it, but it is a way.